Andy Palm N1KSN
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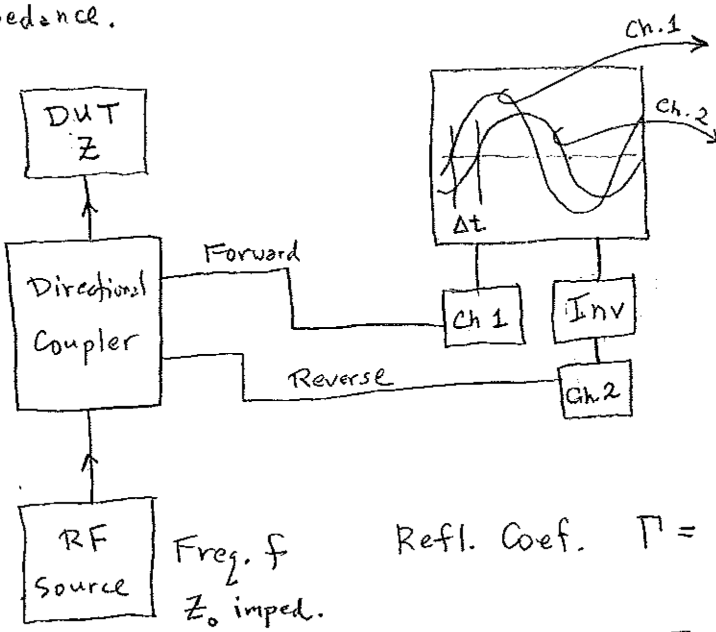
Measuring Impedance with a Directional Coupler

By attaching a directional coupler to a 2-channel o'scope we can measure the (complex) reflection coefficient and hence the impedance.



$|V_f| \angle 0$ (reference phase)

$|V_r| \angle \varphi$ (phase angle φ)

$$\varphi = \pm 2\pi f \cdot \Delta t$$

(+ for V_r leading V_f
− for V_r lagging V_f)

Refl. Coef. $\Gamma = \dfrac{V_r}{V_f} = \dfrac{|V_r|}{|V_f|} \angle \varphi$

$$Z = Z_o \cdot \frac{1+\Gamma}{1-\Gamma}$$ (use complex number arithmetic)

See attached schematic of the directional coupler for more details.

The raw measurements made as above can be corrected (to a certain extent) for systematic errors in the cables, directional coupler, and DUT holding fixture. The correction equation has three (complex) values:

E_D = directivity, E_{RT} = reflection tracking, E_S = source match

calculated from measured Γ for an open ($Z = \infty$, $\Gamma = 1$), a short ($Z = 0$, $\Gamma = -1$) and a matched load ($Z = Z_o$, $\Gamma = 0$).

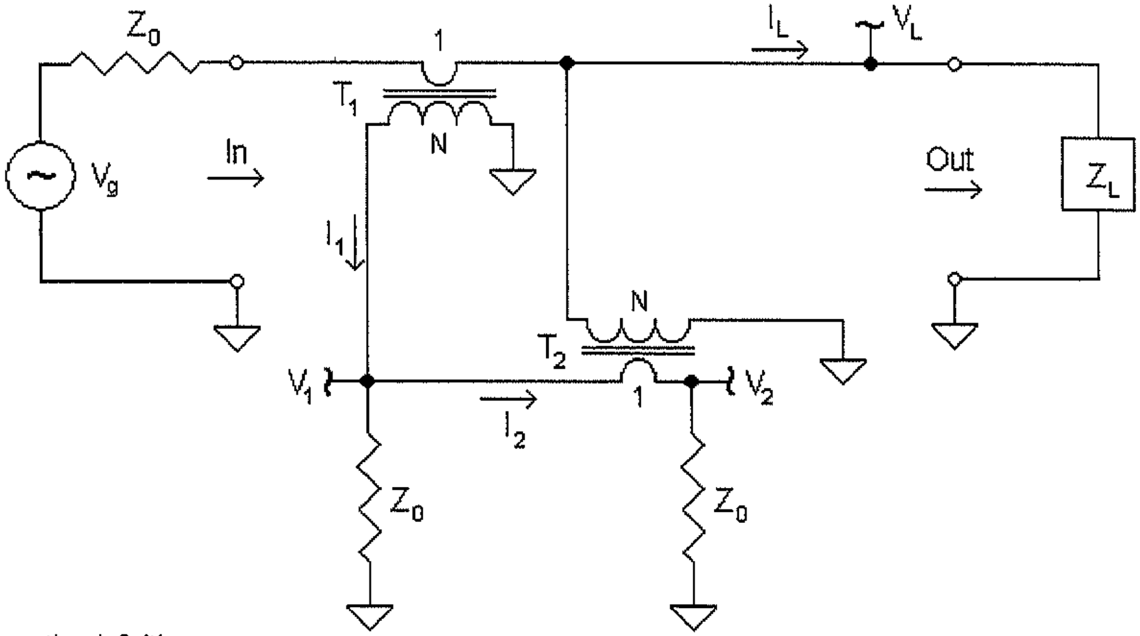
Attached pages contain more details and some examples:

DUT Nominal Value	100 Ω	28 Ω	2.2 nF	100 pF	3.3 μH	15 μH
Adj. Z	98.4 Ω	28.0 Ω	2.14 nF	158 pF*	3.30 μH	15.3 μH

* — Poor est. (high reactive component)

Note: Not all resistive and reactive components are shown, but are negligable, except for 100 pF capacitor.

The Tandem Match Directional Coupler



<u>Definitions</u> (All quantities are phasors)

Reflection Coefficient $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

Forward Voltage $V_f = \dfrac{V_L}{1 + \Gamma}$

Reflected Voltage $V_r = \dfrac{\Gamma V_L}{1 + \Gamma}$

Forward Current $I_f = V_f \ / \ Z_L$

Reflected Current $I_r = -V_r \ / \ Z_L$

If Z_L is the end of a transmission line, then these definitions coincide with the forward and reflected waves on the line.

<u>Assumptions</u>

The voltage drop across the 1-turn primary of T_1 is zero.

The current through the N-turn secondary of T_2 is zero.

From the definitions

$$V_L = V_f + V_r \quad \text{and} \quad I_L = I_f - I_r \quad \text{since} \quad I_L = V_L / Z_L$$

The voltage across the N-turn secondary of T_2 will be V_L, so the voltage across the 1-turn primary is

$$V_1 - V_2 = (1/N)\,V_L = (1/N)\,(V_f + V_r) \qquad (*)$$

The current through the 1-turn primary of T_1 is I_L, so the current through the N-turn secondary is

$$I_1 = (1/N)\,I_L = (1/N)\,(I_f - I_r)$$

We thus get another equation

$$V_1 + V_2 = (I_1 - I_2)\,Z_0 + I_2 Z_0 = I_1 Z_0 = (1/N)\,(I_f - I_r)\,Z_0$$

$$= (1/N)\,(V_f + V_r)\,(Z_0 / Z_L) \qquad (**)$$

Solving the simultaneous equations (*) and (**) for V_1 and V_2 gives

$$V_1 = (1/N)\,V_f \quad \text{and} \quad V_2 = -(1/N)\,V_r$$

Andy Palm N1KSN, Aug. 2006

Freq (MHz): 1 — 1 MHz did a better job than 10 MHz

Z_0 51

Fixture Jaws — alligator clips to BNC male adapter

DUT	$\lvert V_f \rvert$ (mV)	$\lvert V_r \rvert$ (mV)	Δt (ns)	$\lvert V_r \rvert / \lvert V_f \rvert$	ϕ	Measured $\Gamma = (\lvert V_r \rvert / \lvert V_f \rvert) e^{j\phi}$	Measured $Z = Z_0 (1 + \Gamma) / (1 - \Gamma)$
Open	224	244	-32.8	1.08929	-0.2061	1.06623514995121-0.222903504583156j	-175.941779971261-420.471013426654j
Short	333	327	492.5	0.98198	3.0945	-0.980891859196889+4.62576858319917E-002j	0.46389383243539+1.20178222831252j
Match.	270	31.9	-200	0.11815	-1.2566	3.65097856317068E-002-0.112365566184501j	53.4445587464686-12.1806862212212j
100 Ω	254	100.5	-67.15	0.39567	-0.4219	0.360971484103398-0.162030169328327j	98.9752091909042-38.027267853858j
28 Ω	285	70	-501	0.24561	-3.1479	-0.245609186876946+1.54322834241797E-003j	30.8875169748567+0.101453279581757j
2.2 nF	259	294	-210	1.13514	-1.3195	0.282296628673619-1.09947277749736j	-8.53575714858465-65.0523334534211j
100 pF	225	245	-47.95	1.08889	-0.3013	1.03984295518855-0.323118617368963j	-89.3419425769474-310.945697026604j
3.3 µH	295	258	361	0.87458	2.2682	-0.561698930574775+0.670356595785804j	4.15158064626185+23.673721695274j
15 µH	234	223	120	0.95299	0.7540	0.694700871431516+0.652367541120075j	9.02516064874257+128.262621126263j

Nominal values

Measured on scope

$$\frac{|V_r|}{|V_f|} = |\Gamma|$$

$$\Gamma = |\Gamma| e^{j\varphi}$$

$$\varphi = \angle V_r = \angle \Gamma$$

$$\Gamma = |\Gamma| \cos\varphi + j|\Gamma| \sin\varphi$$

These measured values Γ_M of the reflection coefficient must be adjusted for some systematic errors due to imperfections.

$$Z = Z_0 \frac{1+\Gamma}{1-\Gamma}$$

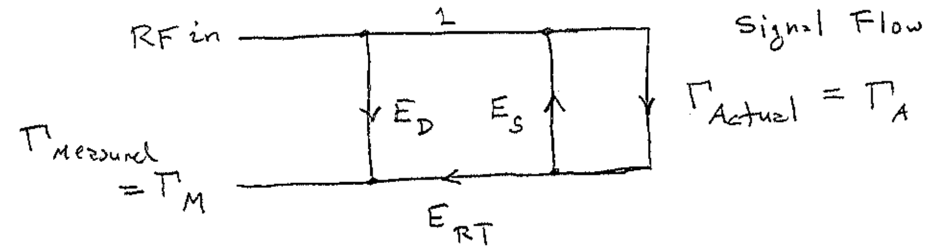
Not adjusted for errors

DUT	Adjusted Γ	Adjusted Z	R (Ω)	C (F)	L (H)	Return Loss (dB)	VSWR
Open	1	Infinite	Infinite			0	Infinite
Short	-1	0	0			0	Infinite
Match	0	51	51			Infinite	1
100 Ω	0.317289520258091-9.06874490039249E-003j	98.3781160815808-1.98425550601946j	**98.38**	8.02E-08		9.967	1.93
28 Ω	-0.286387762090108+7.19880980765342E-002j	28.0442576560812+4.42342965334757j	**28.04**		7.04E-07	10.595	1.84
2.2 nF	0.347301610226974-0.897893984336017j	3.02830709731409-74.3248224396678j	3.03	**2.14E-09**		0.330	52.65
100 pF	0.982774819366333-9.80738374220247E-002j	126.199772352475-1008.91027121942j	126.20	**1.58E-10**		0.108	161.03
3.3 µH	-0.695190616175597+0.678136339335522j	0.869615073132535+20.7496847568634j	0.87		**3.30E-06**	0.254	68.36
15 µH	0.557606823102455+0.823514612802693j	0.636024243488917+96.1204257483279j	0.64		**1.53E-05**	0.048	365.03

O - M 1.0297253643195-0.110537938398655j
O - S 2.0471270091481-0.269161190415148j
M - S 1.0174016448286-0.158623252016493j

E_D 3.65097856317068E-002-0.112365566184501j
E_S 2.88177561549078E-003+2.38680724521074E-002j
E_{RT} 1.02411959935155-0.134796952464542j

Signal Flow for Errors



O = Measured Γ_M for open $Z = \infty$, $\Gamma_A = 1$
S = Measured Γ_M for short $Z = 0$, $\Gamma_A = -1$
M = Measured Γ_M for matched load $Z_0 = 51\Omega$, $\Gamma_A = 0$

$$\Gamma_M = E_D + E_{RT}\left[\frac{\Gamma_A}{1 - E_S\Gamma_A}\right]$$ (complex values)

Solving for Γ_A:

$$E_D = M$$

$$E_S = \frac{(O-M)-(M-S)}{(O-S)}$$

$$E_{RT} = 2(M-S)\left(\frac{O-M}{O-S}\right)$$

Estimated from three known DUTs. Use in correction equation

$$\Gamma_A = \frac{\Gamma_M - E_D}{E_{RT} + E_S(\Gamma_M - E_D)}$$

Correction Equation

Corrected Γ value